Dimensional Analysis
Exercising Problem Solving Skills

OBJECTIVE
Students will become more proficient in solving mathematical science problems.

LEVEL
Chemistry

NATIONAL STANDARDS
UCP.1, UCP.3

CONNECTIONS TO AP
All four AP science courses use problem solving. Dimensional analysis is an essential problem-solving tool and students should be encouraged to practice the skill repeatedly. Some, but not all, of the AP connections are listed below.

AP Chemistry:
III. Reactions B. Stoichiometry 3. Mass and volume relations with emphasis on the mole concept, including empirical formulas and limiting reactants

AP Physics:
I. Newtonian mechanics II. Thermal physics III. Electricity and Magnetism IV. Waves and Optics

AP Environmental Science:
II. Interdependence of Earth's Systems: Fundamental Principles and Concepts A. The Flow of Energy 1. Forms and quality of energy

TIME FRAME
45 minutes

MATERIALS
calculator for each student (scientific, but not necessarily graphing)
student white boards*
dry erase marker for each student*
paper towels*

TEACHER NOTES

***White boards** are the modern version of the old-fashioned slate and chalk. They can be purchased from educational supply houses. However, they are expensive. A much cheaper alternative is to buy shower board from your local do-it-yourself store (Home Depot, Lowe's, etc.) and ask them to cut the board into 16″ × 24″ rectangles. Many stores will not charge for this service if they know that you are a teacher. The white boards can be used in classroom problem solving activities to engage and monitor the students. Students can solve a problem on the board and display it as soon as they get an answer. In that way you can assess each student and spot the students who are not participating or are having difficulty.

Dimensional analysis problem solving is also known as the factor-label method. It relies on conversion factors that are thoroughly labeled with the proper units. Probably the most difficult part of the process is having the students carefully read the problem and write the relationship factors out before beginning to solve the problem.

Here is a sample problem. You may want to use this problem or a similar one to model the technique for the students before asking them to try it on their own.

Example: Susan wants to drive her new car from San Antonio, Texas to her friend Bill's house in Austin, Texas. It is 110. km from her house to Bill's. The speed limit on the highway is 38 m/s, and Susan would never speed. How many hours will it take Susan to reach Bill's house?

Step One: Read the problem carefully. Many students have difficulty with this first step and it is a skill that must be practiced.

Step Two: Determine the unit for the answer. In this case, it is hours.

Step Three: Estimate an answer. This keeps students from making silly errors or calculator errors. Certainly, if the answer came out to 0.0025 hours, the student should know that something was wrong. Certainly it would take more than one hour to go 110. km.

Step Four: Write down all factors and conversions needed under the problem. Make certain that students write the division line in a horizontal fashion and not as a diagonal slash. This seems like a picky thing, but it is the number one stumbling block for many students. For our problem, the factors would be: $110.\,\text{km} \quad \frac{1000\,\text{m}}{1\text{ km}} \quad \frac{38\,\text{m}}{1\text{ s}} \quad \frac{60\,\text{s}}{1\text{ min}} \quad \frac{60\,\text{min}}{1\text{ h}}$

Step Five: Write the unit for the answer at the far right hand side of the page. Precede it with an = sign and draw a straight line backwards across the page. It is recommended to work backwards because it gives students a place to start and better emphasizes the unit cancellation than if they start at the left-hand side of the page.

$$\overline{\hspace{20em}} = \text{h}$$

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Step Six: Since the answer is to be in hours, we need to locate the factor that has this unit. The factor above is $\frac{60\,\text{min}}{1\text{ h}}$. However, the unit hour needs to be in the numerator. We must write the factor as $\frac{1\text{ h}}{60\text{ min}}$. The reason that it is possible to invert the factors as needed often escapes students. You will need to point out that each factor is equal to 1, and that the reciprocal of 1 is still 1.

$$60\text{ min} = 1\text{ hour} \qquad \frac{60\text{ min}}{1\text{ hour}} = \frac{1\text{ hour}}{60\text{ min}} = 1$$

Step Seven: Place the factors in position one at a time, until all that is left is the unit on the left.

$$\frac{110\,\text{k}\cancel{\text{m}}}{} \times \frac{1000\ \cancel{\text{m}}}{1\ \text{k}\cancel{\text{m}}} \times \frac{1\ \cancel{\text{s}}}{38\ \cancel{\text{m}}} \times \frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}} \times \frac{1\text{ h}}{60\ \cancel{\text{min}}} = \text{h}$$

Step Eight: Use a calculator to do the arithmetic. If the number is on the top, the students should use the ↓; if the number is on the bottom, the student should use the ∞. It is best to work from factor to factor, rather than multiplying all the numbers in the numerator and then dividing by all the numbers in the denominator. If you work from factor to factor, the units will tell you what you have solved for at each step. Record the answer with the proper number of significant digits for the problem. Check for reasonableness.

You might notice that the level of difficulty in the student problem set varies between being very easy to much more difficult. This gives students who are having difficulty a chance to be successful, while keeping the interest of students who already have proficiency in the skill. The last problem is about as difficult as chemistry problems get for a first year chemistry course.

You might want to use a song from Michael Offutt's Song Bag II, "Factor-Label It, Baby." It sets a lighter tone and makes a difficult lesson more palatable.

ANSWERS TO EXERCISES

1. The record long jump is 349.5 in. Convert this to meters. There are 2.54 cm in an inch.
   - $\frac{349.5\,\cancel{\text{in}}}{} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{1\text{ m}}{100\ \cancel{\text{cm}}} = 8.877\,\text{m}$

2. A car is traveling 55.0 miles per hour. Convert this to meters per second. One mile is equal to 1.61 km.
   - $\frac{1.61\,\text{k}\cancel{\text{m}}}{1\ \cancel{\text{mile}}} \times \frac{1000\,\text{m}}{1\ \text{k}\cancel{\text{m}}} \times \frac{55.0\,\cancel{\text{mile}}}{1\ \cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{60\,\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\,\text{s}} = 24.6\frac{\text{m}}{\text{s}}$

3. How many mg are there in a 5.00 grain aspirin tablet? 1 grain = 0.00229 oz. There are 454 g/lb. There are 16.0 oz/lb.
   - $\frac{5.00\,\text{grain}}{} \times \frac{0.00229\,\text{oz}}{1.00\,\text{grain}} \times \frac{1\,\text{lb}}{16\,\text{oz}} \times \frac{454\,\text{g}}{1\,\text{lb}} \times \frac{1000\,\text{mg}}{1\,\text{g}} = 325\ \text{mg}$

4. Mercury has a mass density of 13.54 g/mL. How many milliliters would 100. grams occupy?
   - $\frac{100.\,\text{g}}{} \times \frac{1\ \text{mL}}{13.54\,\text{g}} = 7.39\ \text{mL}$

5. In 1980, the US produced 18.4 billion (10^9) pounds of phosphoric acid to be used in the manufacture of fertilizer. The average cost of the acid is \$318/ton. (1 ton = 2000 lb) What was the total value of the phosphoric acid produced?
   - $\frac{18.4 \times 10^9\ \text{lb}}{} \times \frac{1\ \text{ton}}{2000.\ \text{lb}} \times \frac{\$318}{1\ \text{ton}} = \$2.93 \times 10^9$

6. On planet Zizzag, city Astric is 35.0 digs from city Betrek. The latest in teenage transportation is a Zeka which can travel a maximum of 115 millidigs/zip. On Zizzag the planet turns once on its axis each dyne. Their time system divides each dyne into 25.0 zips. How many dynes will it take Pezzi to get from Astric to Betrek to see his girlfriend? There is no telephone communication on Zizzag. Do you think this relationship will last?
   - In this "nonsense" problem, a dyne would be equivalent to a day. Let the students tell you whether the relationship would last or not.
   - $35.0\ \text{dig} \times \frac{1000\ \text{mdig}}{1\ \text{dig}} \times \frac{1\ \text{zip}}{115\ \text{mdig}} \times \frac{1\,\text{dyne}}{25\,\text{zip}} = 12.2\,\text{dyne}$

7. While prospecting in the North Woods, Joe found a gold nugget which had a mass density of 19.2 g/cm^3. When he dropped it into water in a graduated cylinder, the water level increased by 15.0 mL. How many grams of gold did Joe have?
   - $\frac{15.0\ \text{mL}}{} \times \frac{1\ \text{cm}^3}{1\ \text{mL}} \times \frac{19.2\ \text{g}}{\text{cm}^3} = 288\ \text{g}$

8. Light travels at a speed of 3.00×10^{10} cm/s. What is the speed of light in km/h?
   - $\frac{3600\ \text{s}}{1\ \text{h}} \times \frac{3.00\times10^{10}\ \text{cm}}{1\,\text{s}} \times \frac{1\ \text{m}}{100\,\text{cm}} \times \frac{1\,\text{km}}{1000\ \text{m}} = 1.08 \times 10^9\ \frac{\text{km}}{\text{h}}$

TEACHER PAGES

9. A cheetah has been clocked at 112 km/h over a 100-m distance. What is this speed in m/s?
   - $\frac{1\ \cancel{h}}{3600\ s} \times \frac{112\ \cancel{km}}{1\ \cancel{h}} \times \frac{1000\ m}{1\ \cancel{km}} = 31.1 \frac{m}{s}$
   - Note that since multiplication is commutative, students may have their factors in different orders. This frequently happens when the unit for the answer has both a numerator and a denominator. In this case, the answer is in a form that has the first factor number in the denominator. Remind students that they must enter a 1 divided by 3600 to start on their calculator.

10. An electric current of 10.0 amperes is passed through a solution of gold(III) chloride for a period of 0.500 hours. After this period of time, how much gold has plated out on the cathode? There are 96,500 coulombs/mol of electrons. A mol of gold has a mass of 197 g/mol. An ampere is equal to 1 coulomb/second. It is necessary to transfer 3.00 mol electron/mol gold.
    - $\frac{0.500\ \cancel{h}}{} \times \frac{3600\ \cancel{s}}{1\ \cancel{h}} \times \frac{10.0\ \cancel{A}}{} \times \frac{1\ \cancel{C}oulomb}{1\ \cancel{s}\ \cancel{A}} \times \frac{1\ \cancel{mol}\ e^-}{96{,}500\ \cancel{C}oulomb} \times \frac{1\ \cancel{mol}\ Au}{3\ \cancel{mol}\ e^-} \times \frac{197\ g\ Au}{1\ \cancel{mol}\ Au} = 12.2\ g\ Au$
    - Note that in this problem you have a complex factor that is developed from the fact that an ampere is equal to a coulomb/s. You may need to help the students develop this factor. $1\,A = \frac{Coulomb}{s}\ \therefore 1 = \frac{Coulomb}{s\ A}$ The difficulty can be avoided by substituting coulomb/s for ampere before you start working the problem.

T E A C H E R   P A G E S

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Many students have difficulty solving word problems. Your teacher will model a technique called dimensional analysis. You may have learned it in a previous class, so this will just refresh your skill. Even though you may be able to solve a problem easily in your head, you should practice this technique as it will come in useful when problems in chemistry and future science classes get more difficult.

PURPOSE
In this activity you will practice solving problems using the unit cancellation method called dimensional analysis.

MATERIALS
scientific calculator
student white board
dry erase marker
paper towel

PROCEDURE
1. Your teacher will model the problem solving technique for you.

2. Steps for problem solving.
   a. Read the problem. Read the whole problem.
   b. Estimate an answer.
   c. Isolate the unknown with its units.
   d. Write down all the information that is given in correct dimensional analysis format. The division line must be horizontal, not diagonal.
   e. If additional conversion factors are needed, provide them or look them up.
   f. Set up the problem in dimensional analysis format. Work backwards from the unit of your answer.
   g. Use your calculator to do the arithmetic.
   h. Check your answer for units, reasonableness, and significant digits.

3. Work the problem out on the white board. When you have completed the problem, hold it up so that your teacher can see it.

Name ____________________

Period ____________________

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